Observed space is hyperbolic

by

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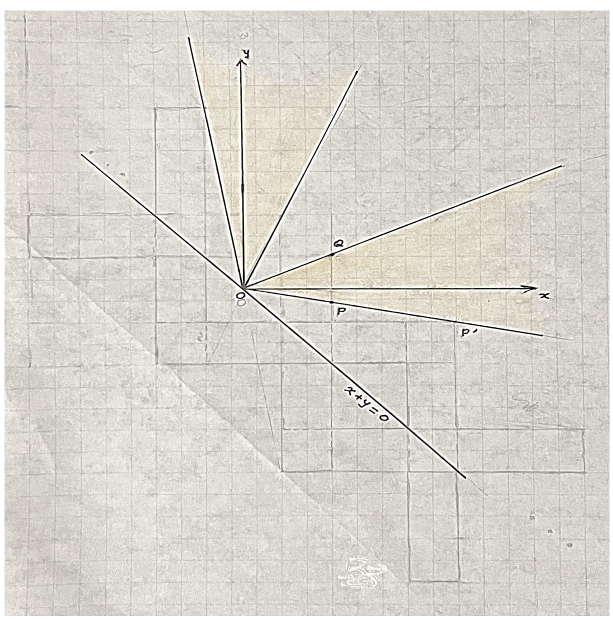
My evidence had been piling up for very long and by 19/08/2025 I was sure that, *this 3-d space we all live in is uniformly curved towards any observer,* and that observed space may well be hyperbolic on the nose.

The assumptions made for the cute proof of *jeanneret jaali*, that I had just posted on 16/08/25, were mainly that all its bricks are identical, with zero (!) thickness of mortar between them. Our theorem holds for any such jaali, of any size, with any number–two or more–of layers of bricks, which is sealed as usual on sides other than front and back. The word *all* in it is crucial: the point from which we view this brickwork runs over all positions.

Also let me add (though we won't need this in the following) that our picture proof yields the two solid *jeanneret cones* precisely:- on the horizontal plane through any O choose x- and y-axes such that the vertical plane $x + y = 0$ is parallel to the jaali; one cone contains all lines through O and the rectangle with vertices $(P, \pm\frac{h}{2}), (Q, \pm\frac{h}{2})$, where $Q = (\frac{l-3w}{4} + w, \frac{l-3w}{4})$ and P on OP', $P' = (\frac{l-3w}{4} + w + \frac{l-3w}{2} + w, -\frac{l-3w}{4})$ has the same x-coordinate; here w, l–we need $3w < l$–and h are the width, length and height of each brick; the second cone is its reflection in the vertical plane $x = y$. From a given O we can see through jaali only in *some* rays of these cones, but as O runs over all positions we exhaust all directions parallel to lines of these two cones because these are *up to parallelism all the lines that can be drawn through the jaali* [1] :



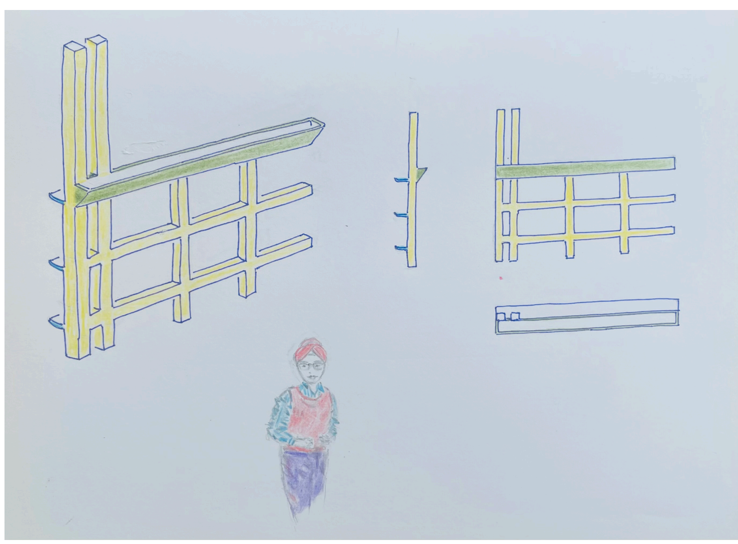
□

[1] Likewise there is, associated to any $\mathbb{R}^n \supset X \supset Y$, the *jeanneret set* $J(X, Y) \subseteq \mathbb{R}P^{n-1}$ of all lines which up to parallelism intersect X but not Y.

The cute proof and theorem had emerged as I recalled how this brick-work was laid (modulo mortar) by drawing four *orthographic views* of two layers: the view from the top or *plan,* and as many as three *elevations*: full frontal, and from the two horizontal directions at 45 degrees to this. More than enough by way of blueprint to make with a child a model of a jeanneret jaali using the identical wooden blocks of a suitable game. Indeed, *an orthogonal projection of an object corresponds to how an eye/camera views it from that direction* only *over an infinitesimal neighbourhood of its nearest point,* and miniaturization is implicit in above usage of the word view.

Or even, to a lesser extent, in most so-called *isometric views,* because though some foreshortening is there, even long parallel lines of say a building–that in a *projective view,* i.e., the intersection with an intervening plane of the cone of rays from the object to an eye, would be depicted as originating from a common point–are drawn parallel. In other words perspective is often not right, unless one is talking only of a model of that building.

Anyway I decided to sketch–this had been long on my to-do list–such a view too, but of *bobby di jaali,* that used to be in our house for many years, and in front of which at least one family group photograph was taken. It was designed by me in late 1959 and built soon after with a very friendly mason whose name (maybe it was Prem Singh) I can't quite remember now:-



Further, even as I was busily wrapping up *jeanneret jaali,* Minni showed me on 12/08/25 an amazing film in colour circa **1960**! Though short and silent it features me, sister Neena, Mama, Papaji, our dog Tiger, and some guests, including an elder cousine, whose son Manohar–not born then–had posted this reel. Its *33 seconds from 65 years ago* [2] shot in 213, 16A–which was 1J, 16A

[2]This video on OtherAuthors shot by me is of the reel playng on Minni's device.

then–show also a much longer jeanneret jaali that used to be on the first floor, and below it: *bobby di jaali* in colour!

The conclusions of this note, though eventually independent, were suggested by the distortion of nearby objects in photos taken in the pano(ramic) mode on my mobile camera. This gives a composite of what its eye sees, as one moves it following an arrow that appears on the viewfinder. A human's roving eye is not so constrained, nor its *horizon* as limited: but luckily–as I had remarked years ago–that curving of the parapet wall, or of the tiling of our rooftop, because of this very limitation, is also evidence that the theory of relativity has kicked in! It will suffice to insert here just one of these photos [3] :-

Indeed, some good old sketching suffices to see that, *there must be a uniform,* albeit very small, *curving of any plane towards the eye,* with the foreshortening of its nearest point depending only on its distance v from the observer, and the distance c to the spherical horizon of our perceptions.

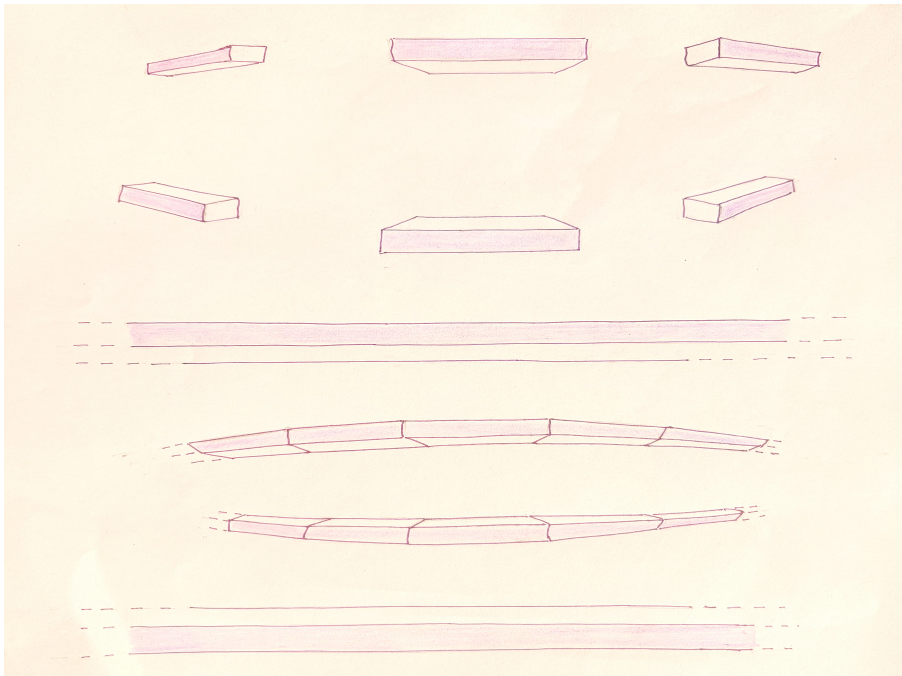
True, an orthographic projection using a family of rays that are parallel, i.e. meeting at infinity, is not right–except infinitesimally when one ray suffices–because the eye is at a finite point, but, even a projective view, using instead the family of rays from the object to this point, can be way off.

To show this I've doodled for starters projective views of six *boxes* from the same point, which is to their front right, centre and left, three above the observer and three below. So far so good, but what if a box is *very* long? To depict in its entirety a *long box,* above or below, we have available only the central view, but then its three visible edges project to three long parallel lines.

So clearly, an eye staring in one direction can't 'take in' a long box, we must let it rove. Say, subdivide this box into small boxes, and fuse the projective views of these boxes, obtained by looking in as many directions: doodled to a smaller scale are also two such *piecewise projective views.* To fit better it seems the pieces 'want' to leave the plane and become facets of a polytope: the

[3] Cf. the photo on p.18 of "213,16A" and mathematics (2010).

lengthwise curving towards the observer of the *long wall* of which these long boxes are the uppermost and lowermost parts is apparent [4] :-



Going to the limit, i.e., subdividing into infinitely many infinitesimally small boxes—the projective view of each is now just the orthographic view in its direction—we see that an eye roving steadily lengthwise will see *a smooth curving of the entire plane* of the pink façade towards it in that direction. More generally, the same argument shows that there is seen a similar and equal curving of the plane *in all directions* parallel to it, because our eye can and will rove in all these directions as well. Finally, using homogeneity of space around an acute 'eye' which can see afar in all directions, all planes at the same positive distance from it must curve alike: in other words, there is a continuous *radial foreshortening of entire space,* only the eye and the horizon stay put.□

However, even for the acutest of observers O, it is common sense that the distance c to the horizon is finite (i.e., for implicit in 'observation' is that it won't take forever–indeed 1 second should suffice–that the maximum speed at which any signal can travel is finite). Hence that pink 'entire plane' is the *open planar disk* of radius $\sqrt{c^2 - v^2}$ around point P nearest to O. How exactly it curves towards O depends on what exactly is the radial foreshortening $P \mapsto P'$ of 'entire space', i.e., the open 3-d ball B^3 of radius c around O. It is natural to ask if *observed radial foreshortening* $P \mapsto P'$ *curves these open planar disks*

[4]Also it recalls: how a human eye sees is more like rotation by 180° of the projection of the cone of rays through a point past it on a *spherical* rather than a plane screen.

around P precisely to open spherical disks around P' normal to horizon, i.e., the 2-sphere S^2 of radius c around the eye O ?

This because, there is a geometrical notion of *reflection in a spherical mirror* generalizing reflection in a plane mirror–the case when the centre of the mirror is at infinity–and a nice *hyperbolic distance* on B^3 preserved by all its reflections in these normal spherical disks and plane disks of radius c and centre O. Much by way of calculations can now easily be done – pp. 26-28 of *"213, 16A"* (2010) suffice for most – to weigh whether the answer to the question above is yes, e.g, an explicit formula for the amount $OP - OP'$ of foreshortening as a fraction of c in terms of v/c, and where it attains its maximum …