## global travel in local time

On 06/07/25, following M & Z's presque-polar geodesic path across multiple time zones, I started musing on some alternative flights. I recall that our globe is the original 24-hour clock:- for any pair of points other than its fixed points, the local time-of-the-day at A is  $4\theta$  minutes ahead of B—because  $4\times360=24\times60$ —if and only if the (semi) longitude through A is  $\theta$  degrees to the east of B; no longitude or time-of-the-day is assigned to the poles  $N, S. \square$  So if B has the same longitude as A flying north or south to it is fine at any speed, but otherwise there is a change in local time-of-the-day of  $4\theta$  minutes irrespective of how we fly. The globe's maximum eastwards rotational speed M at the equator is beyond the reach of today's jetliners, but we can always fly north or south to a latitude, such that on it flying westwards at just the right constant speed S we can easily neutralize rotation, but at this smaller and smaller speed, this now very small hop to the longitude of S, during which we can stop our wrist watch, still takes S0 minutes, after which, restarted, our watch will give the new local time-of-the-day, as we now fly north or south to S1.

For  $s \in (0, M]$  north or south of the *latitudes*  $\pm \phi_s$  on which rotational speed is s, there are also two families of constant speed s westerly spirals that solve our differential equation: latitudinal component neutralizes rotation:- That is, the integral curves of the smooth rotation-invariant vector fields  $V_{s,\pm}$  of length s with longitudinal components always northerly, respectively southerly. These two rotation invariant families of spirals run between the two circles  $\pm \phi_s$  and the poles  $\pm N$ .  $\Box$  Though today's jetliners cruise at speeds below M, we can fly also at any constant speed s > M, when these two rotation invariant families of westerly spirals fill the entire globe minus the poles N, S. In particular there always exists a unique, usually south or north westerly spiral, but occasionally circular, rotation neutralizing flight path that takes us at a suitable constant positive speed from A to B in exactly  $4\theta$  minutes.