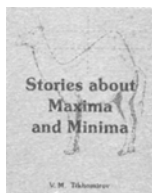


Stories About Maxima and Minima

K S Sarkaria



Stories about Maxima and Minima

V M Tikhomirov

(translated from the Russian by Abe Shenitzer), Universities Press (India) Limited, Hyderabad, 1998.

pp.xi+187, Rs.120

ISBN 81-7371-121-6.

This is an enthusiastically written book which should appeal to our better high school, undergraduate and postgraduate students. Its probable genesis is described as follows on page 135 of the book.

"In the summer of 1984, I coached high school students preparing to take part in the international mathematical olympiad. The topic of the session was 'Mathematical Analysis'. To demonstrate the power of mathematical analysis, I decided to tell the students about topics that you have encountered in Part Two of this book. During the study sessions we arranged a kind of contest between analysis and geometry. I would suggest a problem, the students would solve it geometrically, and I would solve it analytically. I was convinced of the superiority of analysis and hoped for an easy victory. But matters turned out to be anything but simple. My listeners were true lovers of geometry and remarkably well-trained problem solvers. It was a small matter for these youngsters to think of unexpected and very elegant solutions that were – I thought – anything but easy to find. And they regarded them as trivial. There was no easy victory.

The author would like the reader to understand how and why a mathematical theory is born.

Nor could it be said that it was a fiasco for mathematical analysis."

The aforementioned 'Part Two' of this book ranges over topics like Fermat's criterion, Weierstrass' theorem, Lagrange's method of multipliers, calculus of variations, Euler-Lagrange equation, notions like the subdifferential from convex analysis, optimal control theory, etc. Moreover, these topics are not simply pulled out of a hat, but presented after due motivation, and with adequate proofs. Indeed, as the 'Introduction' tells us, "*The author would like the reader to understand how and why a mathematical theory is born*". I think that he has succeeded remarkably well in this stated aim.

The groundwork is laid in 'Part One', in which the author takes us back to the specific problems which eventually gave rise to the above theory – the narration here is very lively, and the historical sidelights are themselves worth the price of this book! – and gives in detail their original, and often very beautiful, geometrical solutions. These include the isoperimetric problem of Dido, Johann Bernoulli's brachistochrone, an algebraical problem of Tartaglia, geometric problems considered by Euclid, Archimedes, Heron and Kepler, the famous inequalities of analysis, and also an aerodynamical problem

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which occurs in Newton’s ‘Principia’. Their original solutions quite often led to important theories, e.g. the optical-mechanical analogy used by Bernoulli in his elegant solution of the brachistochrone problem was the forerunner of the Hamilton–Jacobi theory, and the wave mechanics of de Broglie. The author’s account of these original arguments is careful, pointing out gaps, and simplifying assumptions made, if any.

All these famous extremal problems, and also some others, are later solved anew in ‘Part Two’, in the thirteenth and fourteenth ‘stories’, this time by using the general

methods of analysis mentioned above. The book concludes by some remarks of a pedagogical nature in which the author argues forcefully that “the only way to teach thinking is with concrete ‘special’ problems”. Equally, the compulsion to organize is characteristic of all science: making the right general definition, or weaving an elegant and useful theory out of some known but only vaguely related facts, is also ‘problem-solving’ of the highest order! This book does an admirable job of emphasizing this duality: without its cute and easy-to-state problems mathematics would become boring, without its powerful general theories it would become sterile.

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Addendum

The Indian edition of the book “*Mathematical Analysis, A Straightforward Approach*” by K G Binmore, Cambridge University Press, Second Edition, which was reviewed in *Resonance*, April 1999, p.96 has been published by Foundation Books, New Delhi, 1998.

Acknowledgements

Resonance gratefully acknowledges help received from Jayant Rao and V Pati.