Dear Professor P. K. Jain,

During your talk of February 11 here you mentioned that there exists a Banach frame for ℓ^∞. At my asking if, could you not then give this Banach frame explicitly for this simple space, you had answered no, and gone on to remind everyone how, in various different ways, ℓ^∞ was not simple at all.

However, when I read the definitions again from your paper a few days later, I was rather surprised that it was dead easy to write such a Banach frame; also one has the following general, but quite trivial, result.

THEOREM 1. *A Banach space E admits a Banach frame* (see definition below) *if and only if it is linearly isometric to a Banach space of sequences* (with coordinates continuous).

A *Banach frame* of E consists of a sequence $< f_n >$ of the dual space E^*, an associated Banach space of sequences F such that $< f_n(x) >$ belongs to F for all x in E, a continuous linear map $u : F \to E$ which maps $< f_n(x) >$ back to x; besides, we should have positive constants A and B, such that $A\|x\| \leq \| < f_n(x) > \| \leq B\|x\|$ for all x in E.

Proof. The first inequality shows that $< f_n(x) >$ is a zero sequence iff $x = 0$, i.e., the map $x \mapsto < f_n(x) >$ from E to the vector space of sequences is injective. Assigning to $< f_n(x) >$ the norm of x we see that the image of this injection is an isometric Banach space of sequences.

Conversely, if E is any Banach space of sequences (e.g. ℓ^∞), take $f_n \in E^*$ to be its nth coordinate functional, take $F = E$ and $u =$ identity map, to get a Banach frame with $A = B = 1$. *q.e.d.*

The result above is parallel to the one below, that was highlighted repeatedly in your talk; you had even mentioned that a referee had called it a "breakthrough". The proof below is essentially the same as in your paper, except there was no need to refer to Isaac Singer's book for a rather trivial fact.

THEOREM 2. *A dual Banach space E^* admits a retro Banach frame* (see definition below) *if and only if E is separable.*

A *retro Banach frame* of E^* consists of a sequence x_n of E, an associated Banach space of sequences R such that $< f(x_n) >$ belongs to R for all f in E^*, a continuous linear map $v : R \to E^*$ which maps $< f(x_n) >$ back to f; besides, we should have positive constants A and B, such that $A\|f\| \leq \| < f(x_n) > \| \leq B\|f\|$ for all f in E.

Proof. The first inequality shows that $< f(x_n) >$ is the zero sequence iff $f = 0$. So the rational linear span of $< x_n >$ must be dense in E, otherwise one has a nonzero continuous functional f vanishing on its closure.

Conversely, if $< x_n >$ is dense in E, then $f \mapsto < f(x_n) >$ is an injective function from E^* to the vector space of all sequences. Assigning to $< f(x_n) >$ the same norm as f the image R gives the requisite Banach sequence space, v is the inverse of the injection, and one can take $A = B = 1$. *q.e.d.*

Please be kind enough to check the above. With best wishes, K. S. Sarkaria.

Chandigarh, March 8, 2005.