

Circles and spheres

The present chapter shows how Euclidean geometry, in which lines and planes play a fundamental role, can be extended to *inversive* geometry, in which this role is taken over by circles and spheres. We shall see how the obvious statement, that lines and planes are circles and spheres of infinite radius, can be replaced by the sophisticated statement that lines and planes are those circles and spheres which pass through an "ideal" point, called "the point at infinity." In § 6.9 we shall briefly discuss a still more unusual geometry, called *elliptic*, which is one of the celebrated "non-Euclidean" geometries.

6.1 INVERSION IN A CIRCLE

Can it be that all the great scientists of the past were really playing a game, a game in which the rules are written not by man but by God? . . . When we play, we do not ask why we are playing—we just play. Play serves no moral code except that strange code which, for some unknown reason, imposes itself on the play. . . . You will search in vain through scientific literature for hints of motivation. And as for the strange moral code observed by scientists, what could be stranger than an abstract regard for truth in a world which is full of concealment, deception, and taboos? . . . In submitting to your consideration the idea that the human mind is at its best when playing, I am myself playing, and that makes me feel that what I am saying may have in it an element of truth.

J. L. Synge (1897.)*

All the transformations so far discussed have been similarities, which transform straight lines into straight lines and angles into equal angles. The transformation called *inversion*, which was invented by L. J. Magnus in 1831, is new in one respect but familiar in another: it transforms some

* *Hermathena*, 19 (1958), p. 40; quoted with the editor's permission.

IS THE STUDY OF ITS HISTORY A BRAKE ON THE PROGRESS OF SCIENCE?¹

A LECTURER is expected to say positive things, to impart information of some sort. But I come asking a question: 'Is the study of its history a brake on the progress of science?' I am not going to answer that question. It is no more than a lever to pry open a dark recess which may be in need of light.

The mere asking of such a question suggests that some strange and unusual inquiry is on foot. Commonsense tells us that the question is ridiculous, perverse, impertinent, almost blasphemous. In what possible way could the study of its history hinder the progress of science? What possible harm can come from glancing back down the corridor of time? Does not ordinary human decency demand that we study with respect and admiration the lives and works of the great men whose labours laid the foundations of modern science? It would be monotonous to carry on in this strain. We might wind up the matter by quoting Longfellow:

Lives of great men all remind us
We can make our lives sublime,
And, departing, leave behind us
Footprints in the sands of time.

Commonsense and emotion, hand in hand, shout derision on anyone so foolish as to raise objections against the study of the history of science.

A BRAKE ON THE PROGRESS OF SCIENCE? 21

I am not going to attempt to build up any valid and consistent case against the study of the history of science. But before I proceed to tackle the really difficult matters which may give this lecture some serious content, I might point out a few things.

The history of science is a branch of history. History is supposed to be a narration of facts. But if you keep history down to a mere recital of facts, 1066 and all that—it is a bore. History ceases to be boring only when a certain amount of detail is included, and becomes fascinating only when the detail is so precise that historical characters take on a sort of life in our imaginations. But just at that point, in all the history which I have read, the historian begins to intrude himself. He tries to evaluate history in terms of his own values. Emotive adjectives appear—the historian himself becomes a strand in the history he is writing.

The history of science presents this difficulty, and others too. Sketched in bare outline, it is correct but dull. True, there is dramatic content in the mere statement that Newton (1642-1727) was born in the year in which Galileo (1564-1642) died. But such dramatic datings are rare. When we get into detailed studies, the historian of science, like any other historian, cannot refrain from working himself into the picture. There is a fine book by Professor E. T. Bell, *The Development of Mathematics*. It is a wonderful book, a masterpiece of erudition. But Professor Bell has been unable to keep his own violent personality out of it. His personality, his evaluations in the philosophy of science, shout from the pages. If you read the book fortified by your own philosophy, you can extract the facts and let the rest go. But if you are not so hard-boiled, you may emerge warped by the violence of the author. You may carry away his scale of values.

There is another difficulty about studying the history of science. Science does not consist of a collection of facts. There is of course an enormous collection of facts, but the essence of science lies not in the facts themselves but in

¹ A lecture delivered in the Queen's University, Belfast, 28 November, 1957.

the connections between them. These connections are delicate and highly technical, often mathematical. Any history of science which does not go into these connections is merely superficial. A serious book on the history of science must be essentially technical. It is not a book for the public. If it is recognized that the *history* of science is itself a technical subject, well and good. But one must sound a note of warning against the idea that the history of science provides an easy path into science itself. It does not. The ideas of former ages often seem extremely difficult to the modern technician, let alone the modern layman.

The study of the history of science contains two dangers, which are essentially one. These are the dangers of establishing authority and the danger that science might degenerate into the study of its own history. The whole matter is tied up with ancestor-worship, to which we are prone in ordinary life. It is a dangerous thing to admit ancestor-worship into science.

We have witnessed such thorough revolutions in scientific thought in the present century—I have in mind the theory of relativity and quantum theory—that we may seem in no danger of being cast hard in some conservative mould. But really great figures in science cast long shadows. During the eighteenth century men said: 'That's not the way Newton looked at it!' And to-day men may say: 'That's not how Einstein looked at it!' Authority is built up, and history takes the place of thought.

Yes, there is a danger—though it is slight one at present—that the study of science might become the study of the *history* of science. Is not that true to some considerable extent of philosophy? Would not philosophy be revitalized if *all* philosophical works were burned, and the philosophers had to start from scratch all over again?

But these objections I am raising against the study of the history of science are not very serious—nothing more than semi-audible growls against a civilization nostalgic

for the past. At the present time the history of science is a small neglected subject, and it seems unchivalrous to be nasty to it. But suppose it raised its head? Suppose it grew fat and strong? Like ordinary history?

It is now commonly held that a person cannot become the sort of citizen he ought to be, without studying the history of his country and of the world. In brief, before attempting to live, he must learn how his forefathers lived. Is it possible that the historian of science might make a similar claim? He might assert that a thorough course in the history of science should be an essential pre-requisite to the study of science—that no student should embark on the differential calculus before studying the lives of Newton and Leibniz.

There is something in that suggestion which awakes in me some dark and deep emotion. No one that I know of has really made the proposal that the student of science should approach it through its history, but the mere suggestion of it makes me bare my teeth. It brings to the surface the emotion which lay behind the somewhat perverse title of this lecture.

Perhaps I can sum it up by saying that there is something decadent in preoccupation with the past, that life must be lived in the present, at the very instant at which the future turns into the past. Only in the present can the active mind find its satisfaction. The future is unknowable, the past is dead.

The Greeks were the fathers of our science, and they were great realists, careless of the past. As Oscar Wilde wrote in *De Profundis*: '... they saw that the sea was for the swimmer and the sand for the feet of the runner.' And I would say that any modern schoolboy, wrestling joyously and earnestly with some problem in the geometry of the triangle, is far closer to Euclid than he could possibly get by studying any history of Greek mathematics. One must bring oneself face to face with reality, intellectual reality I have in mind, not seek it at secondhand in history books.

Now I have stuck my neck out. I have denigrated a vast body of scholars whose business is with the past. They are admirable men, and I love them. I hope that they love me. In order that they may go on loving me, it would be well for them to regard what I say as a dialectical jest. For if they sense a note of seriousness in what I say, then they may be tempted to kill me in self-defence. Well, if I am to be killed, it may as well be for a sheep as for a lamb. I want to look round at the academic scene.

It may be that I shall wander away from the title of this lecture. But then, as Lewis Carroll might have said under similar circumstances, the title of the lecture is only what the lecture is *called*. I want to hunt bigger game than the history of science. I want to run to earth, or at any rate to pursue hotly, a certain troublesome thing which is apt to raise its head on a university campus, and which seems to be raising its head higher every day. I may call it the eternal controversy between the ordinary man and the intellectual.

Yes, I want to look round at the academic scene. I want to observe Pallas Athene.

In modern physics much emphasis is laid on the fact that the observation of a phenomenon disturbs the phenomenon. The idea is really not a difficult one. If you observe the presence of a tennis ball by feeling a muscular reaction in your arm, you may be sure that your racket has altered the motion of the ball. This influence of the observer is, however, significant only if the masses of the two bodies involved are comparable. You can use your tennis racket to detect the presence of a motor bus without significantly altering the motion of the bus.

To apply this physical idea to the observation of human affairs, I think I may say that you cannot observe another human being in any deep way without altering him. For to observe him, you have to talk to him, you have to ask questions, and by the time you are finished you have made some impression on him, and he is not the same as he was before.

But I am observing Pallas Athene. She is very large and I am very small. I do not think she will be disturbed by my observation of her. So I crave the liberty of observing the ordinary man and the intellectual, and their struggle at her shrine.

It may not be wise to air this controversy. It may not be wise to refer to it at all. If the dog is asleep, why not let it lie? But I don't think it is really asleep, only shamming sleep, and we might as well wake it up and have a look at it. Certainly I want to have a look at the beast, for it has been growling at me all my life.

I have to go deep, very deep, and I would like to explain my defects as a guide into these depths. It is better to do so beforehand than to leave it to you to find out what will be only too obvious.

I am a mathematical physicist. When I encounter a physical problem, I strip it down to what I consider the bare essentials. Sometimes I strip it down so far that I throw out the essentials; then I have to start all over again. But strip it I must, because physical reality is too complicated to think about coherently with any pretence to logical treatment. This stripping down to essentials is the essence of mathematics. It is true that the mathematician carries out in the course of his work a great many routine operations which could be performed by a well-designed machine, but the reduction to essentials is what makes him a mathematician. When I write books on mathematical subjects for mathematical readers, I try to strip the argument to the bare essentials. I think the reader is more likely to digest it that way. He doesn't need roughage in his diet, if he is a mathematician.

But when I try my hand at a book of the type called 'popular', I tell myself that such an arid treatment would not suit the reader of that sort of book. I go to the other extreme. I wrap up what I have to say in so much brown paper and white paper and red paper that the unfortunate reader never finds out what the parcel really contains. In all modesty, I may claim to be the greatest living

exponent of the art of writing one thing and meaning another.

I mention these literary efforts of mine for two reasons. First, I want to say that, after due reflection, I concluded that the Queen's University would prefer directness to obliquity. Therefore, although everything I say must not be taken quite at its face value (what can?), I try to be as serious as the occasion demands. And secondly, since I am about to accost the ordinary man and the intellectual, I may remark that in a recent book, *Kandelman's Krim*, I depict in my own oblique way the ordinary man and the intellectual. The ordinary man is a simple Plumber. The intellectual, being by his nature inhuman, appears as an Orc—it was only after the book came out that I realized that many people did not know what an Orc was and were so lazy that they would read the book without going to a dictionary to find out what I thought every cross-word-puzzle fan knew.

But now I shall be brutally direct in my definitions of the ordinary man and the intellectual.

The ordinary man, well, he is an ordinary man. He follows Alexander Pope in believing that 'the proper study of mankind is man'. He is interested in men, aye, and in women too, as Shakespeare saith. He is a social creature. He likes talking to men, and in the eighteenth century he frequented places of execution to see men die. Now he reads books about men, being particularly attracted to tales of sex and crime, and he likes the theatre and the movies because they deal with men. Documentary films leave him cold unless they have some human content, and so a love story is usually woven in for his benefit. He does not care for abstraction, and at school he had a particular aversion to algebra. He lives with his fellow-men in amity and goodwill, his religion (when you cut away the theology) being a code of social behaviour designed to that end. But if you scratch him, he will bite. He is the biological reality, ensuring the perpetuation of the race. He is loyal to the core, laying down his life if need be

for the common good. He does not ask many questions except about the affairs of daily life. Any question about ultimate purposes he is likely to brush aside as impious, or leave, with a rather pathetic confidence, to the intellectual to answer. Towards the intellectual, the ordinary man has a feeling of mixed contempt and respect, as towards a clever animal which might turn vicious.

All of which is submitted without malice, and certainly without any attempt to decide whether the ordinary man is a good thing or a bad thing.

The intellectual is much harder to describe. There are so many ordinary men that you might possibly find a man approximating to the description I have given. But it is very hard, indeed impossible, to find in pure unadulterated form the intellectual as I am about to describe him. I am forced to synthesize him by taking bits and pieces from many men I have known or read about.

The intellectual is not much interested in man, neither in the ordinary men nor in other intellectuals. The ordinary man bores him with his gossip, his love-making, his money-making, his football and his horse-racing. And other intellectuals annoy him by looking at things in a way different from his. While the ordinary man is gregarious, the intellectual prefers solitude. To him, man is a trivial incident in the cosmic scheme. He may be kindly and give alms to the poor, but in his heart he thinks of man as trivial incident, even himself. Interesting perhaps as the moon is interesting, but no more central than the moon is central in the solar system.

This picture of the intellectual is, I rather suspect, not quite the picture that the word usually calls up. But I have to define my own terms. To me the intellectual must be a scientist, for only a scientist can appreciate to the full the cosmic triviality of man. Your so-called intellectual author or playwright is no true intellectual in the present sense—he is only an ordinary man gone astray, still thinking about man, even if he thinks pessimistically about him.

It is hard to know whether the intellectual should be

called religious. It all depends on what you mean by the word. He is not religious in the sense that the ordinary man is religious, for the religion of the ordinary man is man-centred and takes faith as its cornerstone. The intellectual is sparing of faith, reserving it for those things which seem, for the time being at least, to be scientifically demonstrated, such as the results of repeatable experiments or natural laws tested many times. But in another sense the intellectual is more religious than the ordinary man, because he is passionately interested in the nature of things. He wants to take everything in pieces, actually or figuratively, in order to discover the plan according to which it was constructed. If an active wonder with regard to the universe is a criterion for religion, then the intellectual is very religious indeed.

You will look for a long time before you find in the flesh the intellectual as I depict him. No adequate statue could ever be put up in his honour, because a statue could show only what the intellectual has in common with the ordinary man. If you demanded a statue, it would have to be symbolic, like the famous statue on the tomb of Archimedes—a sphere and a cylinder. Or the statue might take the form of a university.

I hold the existence of intellectuality to be very real. But existing only in bits and pieces. A one-hundred-per-cent. intellectual might be, probably would be, an intolerable companion. We should be thankful that the type appears only in adulterated form:

Black is his robe from top to toe,
But white and soft the limbs below,
And through his raging veins there flee
Hunger and thirst and venery.

Let me make a fleeting return to the history of science by suggesting a piece of research for the historian of science—let him go over the roll of scientific fame with a red pencil, assigning marks for intellectuality. I believe that

every scientist worthy of the name has in him some dose of what I am calling intellectuality. But the assessment would be difficult. Intellectuality is not to be confounded with mere intelligence, although intelligence of a high order often accompanies intellectuality. No doubt Euclid and Archimedes would score high marks—indeed I wonder whether, in describing intellectuality, I am not groping for a description of the mind of ancient Greece. And what of Newton? Of him Lord Keynes wrote: 'He was the last of the magicians, the last of the Babylonians and Sumerians, the last great mind which looked out on the visible and intellectual world with the same eyes as those who began to build our intellectual inheritance rather less than 10,000 years ago. Isaac Newton, a posthumous child born with no father on Christmas Day, 1642, was the last wonder-child to whom the Magi could do sincere and appropriate homage.' A very complex figure, Newton, but I think Wordsworth labelled him an intellectual when he wrote those famous words:

... a mind for ever
Voyaging through strange seas of thought, alone.

What mark would Laplace score, or Hamilton, or Faraday, or Rutherford, or Einstein? I think that Einstein would rate a high mark; the Jew, like the ancient Greek, is well prepared to think in cosmic terms.

We have before us now our two sets of pieces—ordinary men on one side and intellectuals on the other. Before the game begins, I must warn you that the court will be cleared at the first sign of applause. You must *not* back one side, much as you might like to do so. We are outside the domain of loyalties, except a loyalty to scientific truth, if it can be found.

If the game is played on the board of ordinary life, the intellectual has rather a poor time of it. It lies outside the scope of the ordinary man to understand him, and the intellectual has to pass for an incomprehensible genius or

for the village idiot or perhaps both. But if the game is played on the campus of a university, the outcome is somewhat different. Although the ordinary man may have a quick mind and high intelligence, he has not behind him that force which drives the intellectual, and so in university circles the intellectual enters into his own. There he can live his life as an intellectual—at least he hopes he can. But there are certain pitfalls.

The first pitfall is starvation. If no one gives him money, he cannot buy food. If he gets no food, he dies of starvation. Universities are kind and they do not let their members starve. But the money must come from somewhere. As long as universities lived on their endowments, the danger did not arise. The universities could support their intellectuals and no questions asked. But universities nowadays must rely on public support. The people who guard public funds are ordinary men, or, if not ordinary themselves, hold office by virtue of the votes of ordinary men. Thus, no matter what diplomatic chain there may be in between, the intellectual and the ordinary man become aware of one another. It is in the power of the ordinary man to decide whether the intellectual shall starve, or, if not to starve, whether he shall give up doing what he wants to do and pursue some other calling to earn a living—in other words, turn to some *useful* occupation.

The second pitfall for the intellectual is more an internal matter than an external one, although there may be external pressure. For certain reasons the intellectual permits himself to be transformed into an ordinary man, a very important ordinary man, a responsible ordinary man, but still an ordinary man. Perhaps his inner nature does not change, he may remain an intellectual at heart, but the pressure of responsibility will prevent him from pursuing the intellectual life.

If I were E. T. Bell I would probably at this point enlarge with horror and indignation on the loss to science occasioned by Newton's acceptance of the appointment of Master of the Mint. I want to make it quite clear that

I am trying to conduct this inquiry on a more scientific plane, above the level of praise or blame. I am trying to analyze the world as it is, without seeking to improve it. Perhaps it needs no improvement. In any case, improvement means nothing at all except in terms of a scale of values. I lay before you not one scale of values but two, the scale of values of the ordinary man and the scale of values of the intellectual. They are different, and if you improve things according to one scale, you are likely to make them worse according to the other.

When I see a first-class scientist translated into an administrative position, I am both glad and sorry. As an ordinary man I am glad, because administration must be done and done well, and I have confidence that the scientist's intellectuality will not hinder his administrative capacity. But as an intellectual I am sorry, because when I said that intellectuals do not like other intellectuals, I did not really mean it. To an intellectual, a second intellectual, annoying as he may be through divergence of views, is more exciting company than an ordinary man.

The motives which move an intellectual to act as an ordinary man are many. He will probably earn more money and have more power, but I am inclined to think that the reason lies deeper. He wants to be *useful*.

I have used the word *useful* twice, and I underlined it in my script to emphasize it. It is the key word. What does *useful* mean?

In a scholarly history of science no doubt all apocryphal anecdotes will be eliminated. But while we remember them, we may use them. There is one about Euclid—how one of his students asked him whether he could make money out of geometry, whereupon Euclid in some contempt instructed his slave to hand the student a small coin, seeing that that was what he was seeking. There is a somewhat similar anecdote about Newton. In spite of all that Francis Bacon wrote, I believe that until comparatively recent times, say a hundred years ago, few scientists would have regarded the word *useful* as applicable to their work.

But times have changed. There are many scientists now who think that, although their research may not be immediately useful, it is basic science which, before long, will yield useful results. Even the purest of mathematics may be applied to industry before very long. You cannot draw the line anywhere. And if Euclid and Newton did scoff at utility, how useful their work now appears! Where would modern technology be without geometry, without dynamics, without the differential calculus?

But what does the word *useful* mean? Like most ordinary words, its meaning is to be found in the dictionary of the ordinary man. The ordinary man certainly regards medical research as useful in that it promotes health and prolongs life. The ordinary man regards rapid travel as useful because it gives him a longer life in a sense—he does not waste his life in travelling. Everything that promotes life is useful to the ordinary man, and that includes all industrial developments which lead to increased employment and hence increased population. In fact, what the ordinary man means by *useful* is *biologically useful*.

There is what is called a tradition of learning in these islands. That does not mean that there are really many learned men—the percentage attending universities is not high—but there is a certain respect for learning. In terms of the discussion I am trying to pursue, this means that the ordinary man is prepared to respect the intellectual and even to support him in his intellectual activities without demanding *biological usefulness*. But in the newer countries things are a little different, particularly in the state-controlled universities. I was on the staff at the University of Toronto, and later on the staff of the Ohio State University, and I could feel, particularly in the latter, an unanswered question hovering in the air: *Is what you are doing biologically useful?* I think that this question is going to be asked more frequently in future on this side of the Atlantic.

It is going to be asked more frequently throughout

Western Europe. We know the ideology of Western Europe fairly well. Whether the question is asked in Moscow I do not know. I am told that the Russians treat their scientists with great respect, but until the Iron Curtain is removed we cannot tell whether this is because the scientists have accepted the idea that they should be biologically useful. I do not know whether the core of the Communist Party consists of ordinary men or intellectuals.

As we gaze down from our eminence on the animal kingdom and the insect world, and note how important the biological urge is, we may be led to some rather uncomfortable thoughts—uncomfortable, that is, for those of us who treasure the idea that we are individuals who can make decisions for ourselves. Perhaps I am only the tail of the dog, and I am deluded in thinking that I wag myself. Perhaps everything we do is dictated by biology, by the race, by the Life Force. Perhaps I did not really prepare this lecture for myself. Perhaps the Life Force dictated it to me as the very thing needed in Belfast at the present moment. If so, all I can say is that the Life Force is not much good at dictation—otherwise I would have got it written out much more easily. But perhaps my slowness was due to a struggle against the Life Force; perhaps I was making a pitiable effort to say things the Life Force did not like.

No, I refuse to believe it. I stick to the old-fashioned idea of individualism. I claim the power to make decisions, good or bad, for myself. I refuse to accept that everything we do must of necessity be biologically useful.

But I am getting into deep water and must swim for the shore. If I had not sworn myself to complete impartiality in this discussion—if I permitted myself to take the side of the intellectuals—I would say that the institutions of learning are in danger. To be precise, they are in danger of being asked whether what they do is biologically useful. In view of the almost unlimited applicability of basic scientific research, I think that the universities can honestly answer *Yes*. Whether the same

affirmative answer can be given with regard to non-scientific subjects, I do not know. Those engaged in the subjects usually called cultural must fight their own battle—I wish them luck.

But the matter does not end with an official answer given by the universities to those ordinary men who control funds. There are internal subtleties, no less real for being subtle. I have in mind the *conscience of the intellectual*. It is all very well to say that what he is doing is biologically useful. You have to consider his own conscience, and the delicate balance between the intellect and the conscience.

At this point I feel that I must talk about myself. I must tell you of my own experiences, since it is of them alone that I have any true knowledge. But first let us be clear as to our terms.

You must understand that *biologically useful* means useful in the ordinary sense—I insert the adjective *biological* merely to stress that this ordinary usefulness has, somewhere in it, a biological content—something to do with the health of humanity, with convenience of living, with the propagation of the species. I have given a shadow picture of the intellectual, and indirectly of the intellectualism for which he stands. Can I make the concept of intellectualism any clearer? I feel that I must try. The solution of a chess problem may resemble very closely the solution of a scientific problem. Yet I would rather not class chess as part of intellectualism. Why not? Because it does not seem to bear at all on our knowledge of the universe. It has no cosmical significance. True, the cosmical significance of some trivial scientific problem may be small, but I think it is there nevertheless. Everything ties up in the end.

Nor do I include in intellectualism the arts of literature, architecture, painting, sculpture, music. These differ from science in that works of art may be publicly exhibited and, at least to some small extent, understood and appreciated by those who are not practitioners of the art involved. I do not think that the inner significance of scientific work can be similarly appreciated by those who are not themselves

scientists. I want to keep the word intellect for science alone.

If I have cleared the air a little now, let me turn to my own experiences. Throughout my scientific life, say forty years, I have felt a schizophrenia—a splitting of the mind—in the matter of biological usefulness and intellectuality. If the one dominated my conscious mind at a certain time, the other ruled the subconscious, and they took turn and turn about.

When I was an undergraduate, I threw myself upon mathematics and physics in an intellectual spirit. But on the conscious level I believed in biological utility, and I remember shocking the mathematician-philosopher R. A. P. Rogers in Trinity by asking him what was the use of analytical geometry of three dimensions. I forgot with what answer he brushed me off—it was clearly a question bordering on blasphemy.

For many years I taught mechanics and other branches of applied mathematics. I felt I ought to bring in *useful* applications and I tried to do this, but it was never a success. What really interested me was the intellectual side of the subjects, their thought-structure and how they were connected with one another.

While in Toronto, I was Professor of Applied Mathematics, and I deplored the almost complete lack of interest in applied mathematics all over the North-American Continent. Such vital subjects as hydrodynamics, for example, attracted no attention whatever. The last war changed all that. I helped in the propaganda for applied mathematics at the beginning of the war, and in due course was Chairman of a new committee for applied mathematics set up by the American Mathematical Society. But, curiously enough, as soon as this had been set on foot, I lost interest in it, because the subject seemed to be losing its intellectuality and becoming biologically useful.

The war widened the scope of the biologically useful. It was as biologically useful to kill others as to keep from being killed oneself. By that time I had reached sufficient

maturity to recognize some of the complexity of human nature. I realized that there was a tension in me between the biologically useful and the intellectual. But I thought now, if ever, is the time to be biologically useful. Nevertheless, to be honest, I think that with all my effort in this direction I did very little. I was more concerned to *understand* than to *do*.

One little thing I did. In the first world war, a group of scientists, led by R. H. Fowler, did some important work (experimental and theoretical) on the flight of projectiles. In 1942, in connection with some lectures I was giving on dynamics, I played a little with the problem of a spinning projectile, and thought that Fowler had approached the problem in a rather clumsy way. I tried a different approach, and came to the conclusion that Fowler had failed to obtain one of the components of the rather complicated force-system which acts on the projectile due to the resistance of the air. This discovery² in due course reached the military authorities in the United States, and largely on the strength of it I came to Europe for a year as a scientist attached to the American Air Force, and subsequently used to act as consultant at Aberdeen Proving Ground in Maryland.

But the joke of all this is that the force which I discovered is extremely small in practice, and its biological usefulness practically nil. My experimental friends at the Aberdeen Proving Ground used to try to cheer me up by indicating how occasionally a firing gave evidence of this tiny force. I pretended to be quite excited, but at heart was not. The satisfaction which I took in the discovery was not on the plane of biological utility, but on the intellectual plane. I felt that I had handled the problem in a much deeper and pleasanter way than Fowler had, and that was the real source of any satisfaction I felt.

But who knows? It may be that for some future

² Worked out in detail with the collaboration of Dr. K. L. Nielsen, and published later (*Quarterly of Applied Mathematics*, iv (1946), 201-226).

projectile this little force may become biologically important. It may enable someone to kill someone else.

In my present position in the School of Theoretical Physics of the Dublin Institute for Advanced Studies, I would rather not be asked the question: Is the work of the School biologically useful? I would not like to say *No* because I might be telling a lie. For although a piece of work may not seem to be of any immediate biological use, it is impossible to say what will, or will not, prove useful at a later date. I would go so far as to say that the immediate motivation of our work is not biological utility. The immediate motivation is intellectual. And I have a very strong suspicion that throughout the world, even in Russia, the real motivation behind scientific research of truly basic character is intellectual, not utilitarian. If the ordinary man is worried about spending his money in this way, he may be told that you can lead a horse to the water but you cannot make him drink unless he is thirsty—a proverb which should be posted on the walls of all classrooms and lecture theatres throughout the world.

Some years ago I had to write a review of Sir Edmund Whittaker's book on the History of the Theories of the Aether. The book is an excellent example of the art of the historian of science, although (to repeat what I said earlier about the history of science in general) one should be a mathematical physicist in order to read the book. At that time I was already engaged in that long quest of which this lecture is part, and I noted that Whittaker gave no indication of the impelling motive which urged so many men through so many years to devote themselves to the unravelling of electromagnetism. To this big question—the question of motivation in scientific work—I would like to devote the rest of this lecture.

What makes the scientist tick? In seeking an answer, I do not think we should recognize any essential difference between those scientists we call great and those we call minor, or between the minor scientist and the schoolboy or undergraduate pursuing the study of science.

What sort of an answer could be made to such a question? Well, we might start by suggesting that a scientist does his work in order to make money, immediately or later. There is a modicum of truth in that. Everyone must live. And a young man may be spurred on to research to gain some academic position—a doctorate is becoming increasingly a *sine qua non*.

Then there is honour and glory. What did Milton say in *Lycidas*?

Fame is the spur that the clear spirit doth raise
(That last infirmity of Noble mind)
To scorn delights, and live laborious days; . . .

I suppose that there must be something in this too. And yet I think it can be over-emphasised, and this over-emphasis leads to an over-emphasis on examinations. After all, examinations are a modern invention, and so are Nobel Prizes. There was a lot of fundamental science done before their time.

Perhaps I am a romantic, but I feel that neither the desire for money nor the desire for glory touch the heart of the matter. I have been a teacher for many years, and have known students of all degrees of competence. It is good for the conscience to have explained something rather simple to some students who are rather dull. It is a real thrill to encounter that thing which I am calling intellectualism. Something unusual stirs. Some fire is lighted. The ordinary values of life are overturned. This problem, the understanding of this situation or that, has suddenly become the only thing that matters. It is a sort of possession by a demon. I have known this possession myself, and have many times seen it in students and in colleagues. It is rather spoiled by rewards or praise. Rewards and praise are properly for conduct which is biologically useful, for hard and painful work done for others, for sacrifice. This intoxication needs no reward—it is an end in itself. If there exists such a thing as an ordinary man as I depicted

him earlier, I do not think he ever felt this urge, this possession. Other powerful urges possess him at time, but not this one. If there should be any ordinary man or woman present here, they will think, I am sure, that I am talking through my hat. However, let us never forget the words of the old Quaker to his wife: 'The whole world is mad except thee and me, and I am not sure about thee.'

This lecture has involved a terrible lot of introspection, which is tiring to do and not very interesting to others particularly those whose heads are different. Let us have a look at the animal kingdom. Mankind seems to be coming round to the view that the difference between man and the lower forms of life is less than used to be supposed. In the behaviour of animals, birds, and even insects, free as they are of self-consciousness, man may see himself in microcosm.

Even if you refuse to accept the ape as a sort of man, I would recommend to you a fascinating book by Köhler on the *Mentality of Apes*. It is the fruit of a close study of the behaviour of chimpanzees in Tencriffe over a period of six years. Later Köhler moved into human society, being Director of the Psychological Institute at the University of Berlin, and for twenty years Professor of Psychology at Swarthmore College. Köhler describes most vividly how his apes solved mechanical problems presented to them—the initial inertia, the groping, and finally the sudden rush of understanding of the complete solution. I recognized in this description a valid account of the way in which at least one mathematical physicist solves his problems. The only thing written about humans to which I can compare it is Hadamard's book on the *Psychology of Invention in the Mathematical Field*.

But Köhler's apes were offered bananas out of reach as the incentive for their efforts, a Nobel Prize for chimpanzees, and I have already said that, although reward may mean something, we must look further for the explanation of the real motivation behind scientific research. I have in mind here research of a deep and basic character.

Perhaps we should have another word—*research* does not seem to be the right word to describe the thought by which Einstein, at one sweep, abolished the concept of Newtonian time from physics. I was at a loss what name to give to the basic motivation leading to thoughts of such a deep scientific character, or even to the lesser thoughts which come to any scientist in the course of his work. True, I have suggested the word *intellectualism*, but it hardly seems to ring the bell. I felt I would have to end this lecture without saying *the word*.

However, the birds of the air came to my assistance, offering an explanation, a word, which may strike you as a ludicrous description of such a serious and holy thing as scientific thought. I was watching the seagulls and jackdaws soaring in the wind behind my house against a blue sky. Now I was brought up to think that birds and animals, and men too, must spend nearly all their lives in the pursuit of food and the other things necessary to maintain life—our old friend the biological necessity again. But those seagulls and jackdaws did not *seem* to be looking for food. Perhaps they were saying to one another: 'Yes, we are looking for food; what we are doing is biologically useful.' But I don't think they were looking for food. They were playing, playing with the wind, playing with their strength of wing and delicate sense of balance. And the thought struck me: Why do I not play? Why do I have to sit over my books and papers as I do? Suddenly the thought came to me like a revelation: *I am playing*. My whole life has been playing, playing with ideas. This intellectual element in man, which I have tried to describe and which probably sounded romantically grim—why, it is nothing but an irrepressible desire to play. There may not be so much difference after all between chess and science, except that the moves in science are so much more varied. Can it be that all the great scientists of the past were really playing a game, a game in which the rules are written not by man but by God? That is a thought I would like to develop in the few remaining minutes of this lecture.

A lecture fulfils two purposes. Those who listen to it may perhaps gain something from it; the man who makes it up certainly does gain something, because he has to put into articulate form thoughts which otherwise might remain inarticulate. When I began to prepare this lecture, I did not think I would end it by stating that there is an element of play in all significant scientific discoveries. But does that statement mean anything? What is *play*?

There was a time in our lives when we knew what play was—up to the age of puberty, when the serious biological urge began to take possession of us. Most of us then, following the words of St. Paul, began to put away childish things. Our games were organized in order that by playing them we might learn the biologically useful art of coöperation. But some little pockets in our minds, the intellectual pockets, remained safe from St. Paul, and in those pockets the play went on. True play is essentially irresponsible. It may be, it often is, destructive. When we play, we do not ask *why* we are playing—we just play. Play serves no moral code except that strange code which, for some unknown reason, imposes itself on the play.

All these features may be detected in scientific thought, particularly in profound thought. The scientist does not really care about the biological value of his thought—he is irresponsible in the social sense. He destroys—as Newton destroyed the mechanics of Aristotle and as Einstein destroyed the mechanics of Newton. He never asks why he does what he does—you will search in vain through scientific literature for hints of motivation. And as for the strange moral code observed by scientists, what could be stranger than an abstract regard for truth in a world which is full of concealment, deception, and taboos imposed on us by our tailors and others?

I do not wish to imply that the whole work of a scientist is merely play. I recall words spoken by the late Professor H. N. Russell, the Princeton astronomer: the pursuit of an idea is as exciting as the pursuit of a whale, but when the problem has been solved, the work of writing up your paper is as hard and monotonous as cutting up blubber.

But I do think that, in every piece of scientific work of any importance, there comes moment when the mind has to grapple with baffling difficulties, and that it will grapple successfully with those difficulties only if it plays with them. It will not succeed if it allows itself to dwell on the prospect of reward for success or of blame for failure. It must have that light-hearted irresponsibility which is the essence of play. In submitting to your consideration the idea that the human mind is at its best when playing, I am myself playing, and that makes me feel that what I am saying may have in it an element of truth. Let the historian of science open his books again and read between the lines, asking himself whether the ponderous seriousness of scientists is not but a veil for minds at play. Let the historian ponder in particular over certain words which Newton wrote when an old man. Let the historian reflect that Newton died in 1727, before the invention of that mock-modesty, that mock-humility, which has marked later ages. Let the historian open his mind to the possibility that Newton may actually have meant to say what he said in these words:

I know not what the world will think of my labours, but to myself it seems that I have been but as a child playing on the sea-shore; now finding some pebble rather more polished, and now some shell more agreeably variegated than another, while the great ocean of truth extended itself unexplored before me.

J. L. SYNGE.